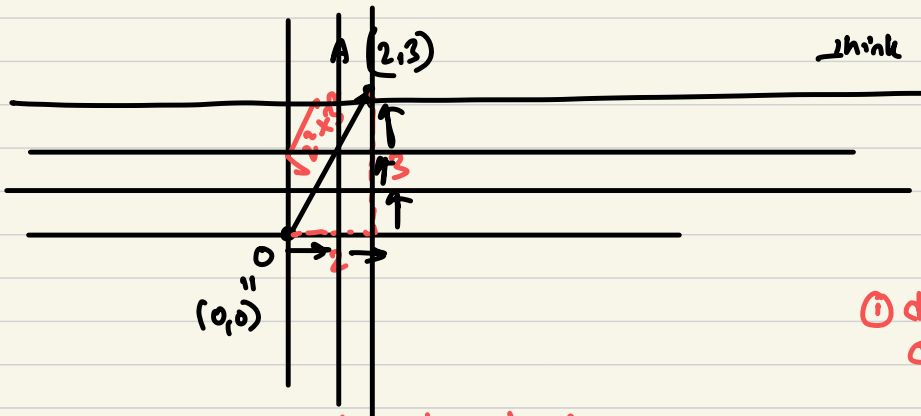
DAY 3 → Tool to solve Geometry with Algebra

Main idea

Think of points as numbers → pair of numbers.
↓
Coordinates.



think of points as arrows.
$\overrightarrow{OA}$ (vector)
① direction O → A
② how far is A from origin ← length of the segment OA → magnitude

Other ways of calculating distance

Taxicab method → Manhattan method.

RRUUU
RUURU

Route

shortest paths on the grid.

How many such path possible from O to A?

bijection

number of paths

number ways of writing code

All the ways you write.

R's (2 R's) U's (3 U's)

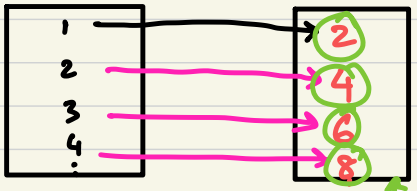
Surjection

no two numbers are pointing at the same number.

bijection = injection (one on one) + surjection (onto) mapping. (correspondence between)

natural

even



1's mapped to 2

1 input → double the input → 2 output

target set (codomain)

add 1 to the input

onto! every number in target set has some input pointing to it

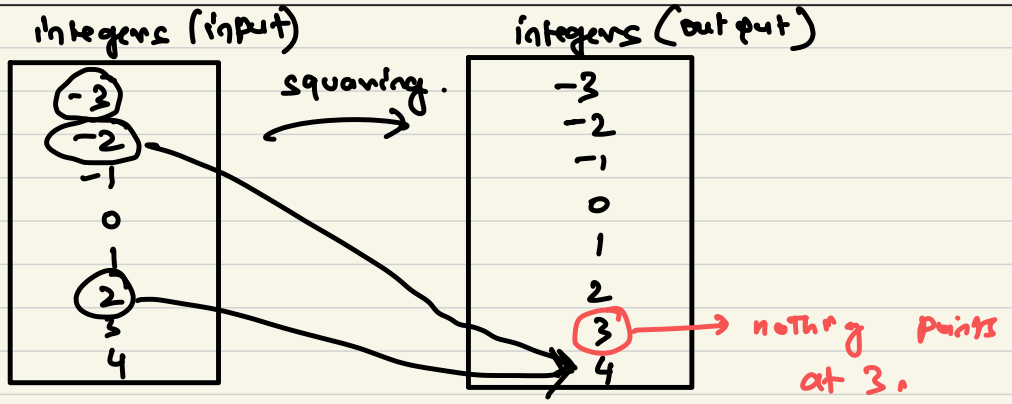
$(-3)^2 = 9$

$(-2)^2 = 4$

written

one on one,

not onto

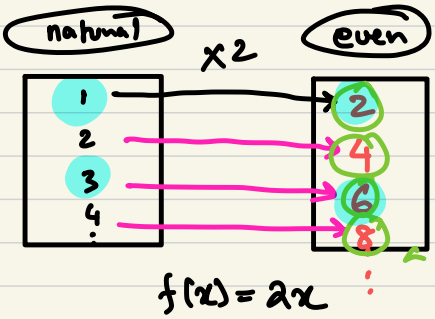


Cantor created some weird mathematics

Question how can we compare "number of elements" in two sets?

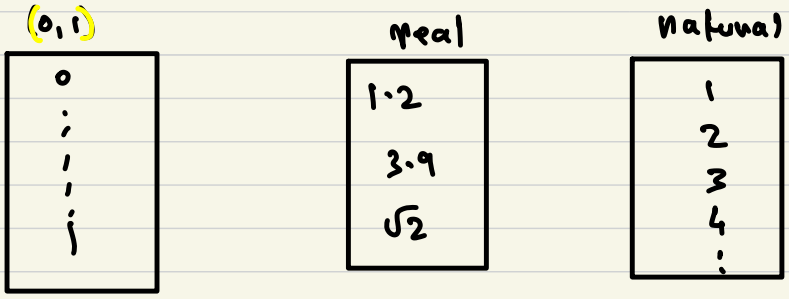
definition two sets have **same cardinality** if there is **bijection** between them

intuition gives way to reason.



natural = # even.

Cantor's diagonalization argument.



much harder

no bijection between these two sets.

natural → real

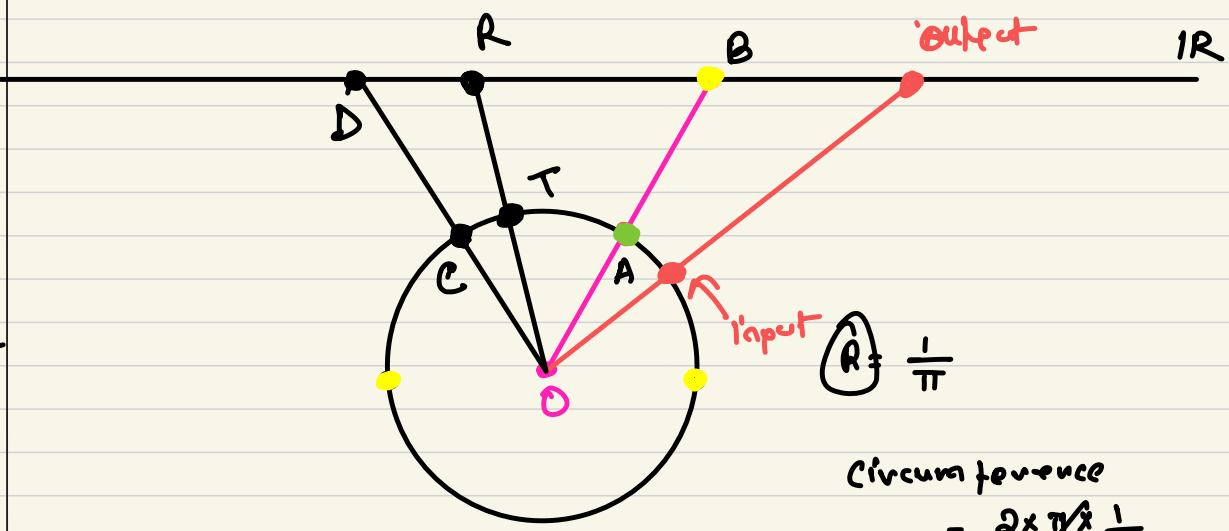
↑

one on one map

Injection

Geometric way of creating a function.

semicircle → $\mathbb{R}$



What happens to every input

Circumference $= 2 \times \pi \times \frac{1}{\pi} = 2$

Semicircle = 1

Homework: make formula for this geometric function.

input : (x, y) (a coordinate on the semicircle)

output : (x_1, y_1) (coordinate of the output point on the line)

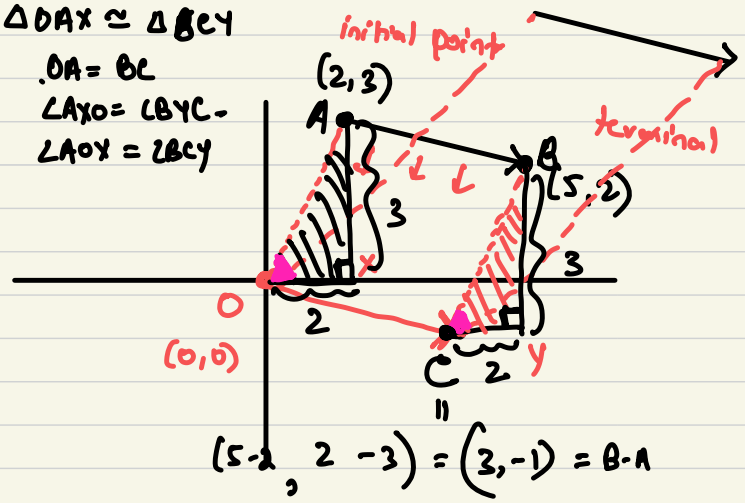
Convert segments into points

$\triangle OAX \cong \triangle BCY$

$OA = BC$

$\angle AXO = \angle BYC$

$\angle AOX = \angle BCY$



Representation using a point

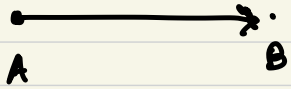
① Loss of information

② Gain → a point to think about an area.

OABC → if you can show that one pair of opp. sides are parallel and equal

automatically the other pair is also //

Conclusion

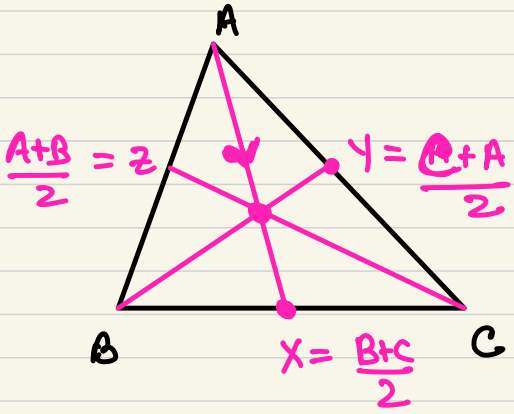


represented by.

⊙ (B-A) [final − initial]

coordinate wise subtraction.

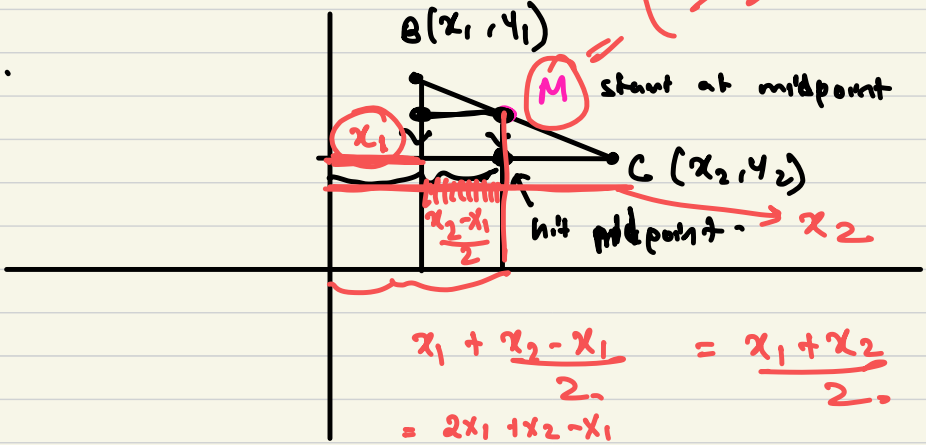
Reminder



① Midpoint formula. $= \frac{B+A}{2}$

② Triangle condition.

Converse of midpoint Theorem



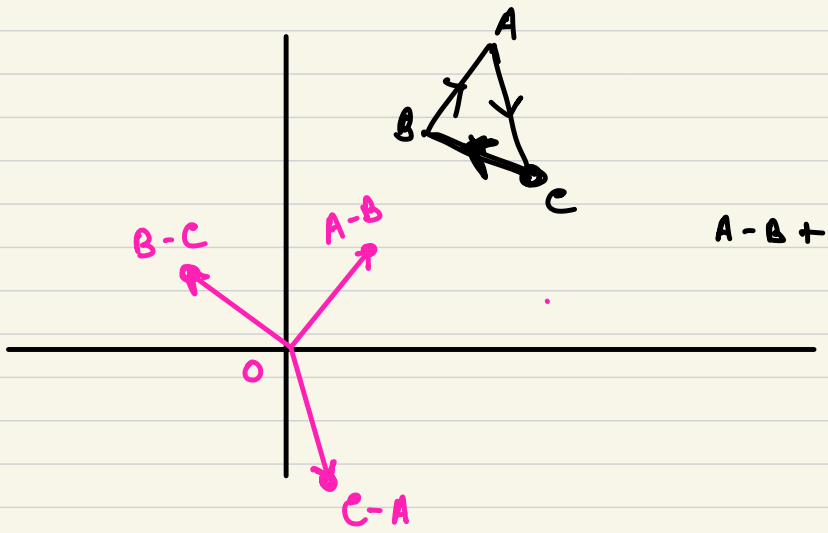
$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$x_1 + \frac{x_2 - x_1}{2} = \frac{x_1 + x_2}{2}$

$= \frac{2x_1 + x_2 - x_1}{2}$

$= \frac{x_1 + x_2}{2}$

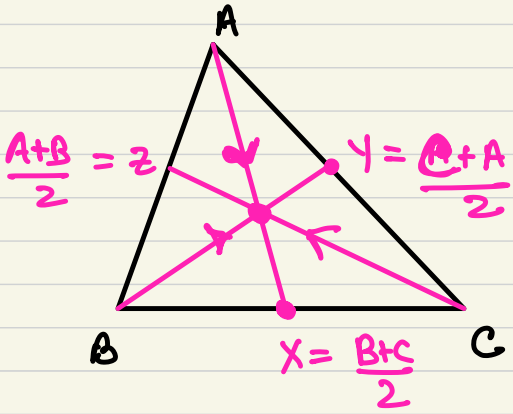
(final - initial)



$A-B+C-A+B-C=0$

Reverse logic

If we add all the representatives and 0 then the original arrows must have formed a Δ.



$\vec{AX}$ to origin $X-A=\frac{B+C}{2}-A$

$\vec{BY}$ to origin $Y-B=\frac{C+A}{2}-B$

$\vec{CZ}$ to origin $Z-C=\frac{A+B}{2}-C$

$\frac{B}{2}+\frac{C}{2}-A+\frac{C}{2}+\frac{A}{2}-B$

$+\frac{A}{2}+\frac{B}{2}-C=0$