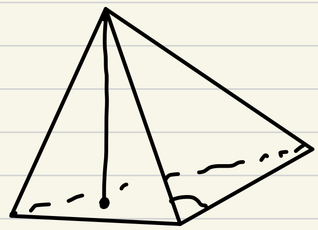
DAY 1

Synthetic + Transformation



how a certain shape should look

What are the details (length, angles)

Felix Klein

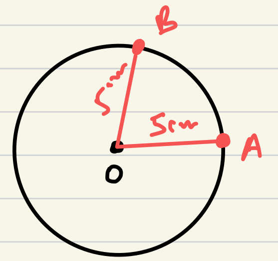
different definition of geometry

set of points $\{P_1, P_2, \ldots\}$

set of functions ← transformations

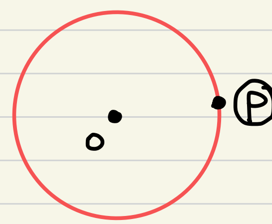
changing this set of points.

Example



GREEK MANNER

describing a shape.



Klein.

point P is acted upon by a rotation function → output a point

↑ inputs a point

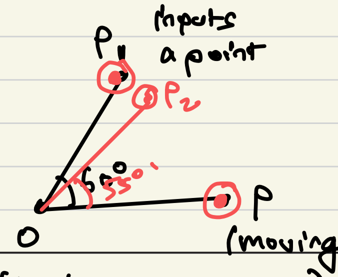
function is an input output machine

$4 \leftarrow$ $\boxed{\text{Squaring}}$ $\leftarrow 2$

$9 \leftarrow$ $\leftarrow 3$

two dimensional things

$P' \leftarrow$ $\boxed{\text{Rotate input point by } 60° \text{ about O}}$ $\leftarrow P$



$\text{Rotation}_{55}(P)$

$\text{Rotation}_{60°}(P)$

[Cornell method]

main topic

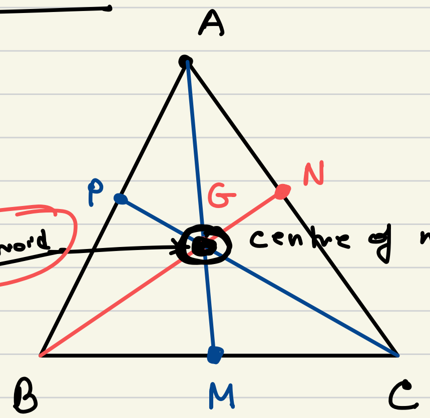
↳ Synthetic Geometry

Triangles

[lot of proofs]

median

AM is the median.



Centroid

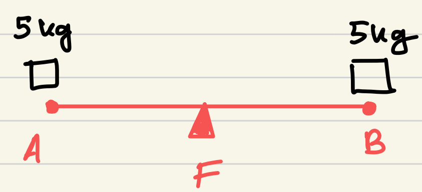
centre of mass

All three medians of triangle pass through same point.

(did not prove this)

Mass point technique ~~of thinking~~ about geometry

[Preview of the centre of mass technique] [Archimedes principle]

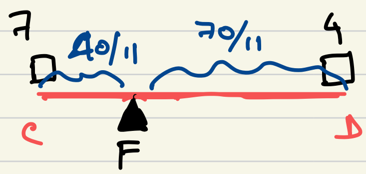


see-saw

calculation question

CD = 10cm

location F ?



$7 \times CF = 4 \times FD$

$7 \times CF = 4 \times FD.$

$1 + \frac{7}{4} = \frac{FD}{CF} + 1$

$\Rightarrow \frac{11}{4} = \frac{FD+CF}{CF}$

$\Rightarrow \frac{11}{4} = \frac{10}{CF}$

$\Rightarrow CF = \frac{40}{11}$

200 m

0

100 m

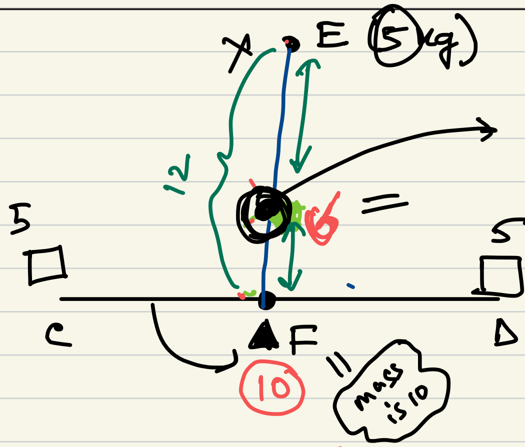
100 m

acceleration

[Summary]

$\frac{2}{1}$ 6 and 3

$\frac{6}{3} = \frac{2}{1}$



this point is our old friend

$\frac{EG}{FG} = \frac{10}{5} = \frac{2}{1}$

$FG \times 10 = EG \times 5$

$\frac{8}{4} = \frac{2}{1}$

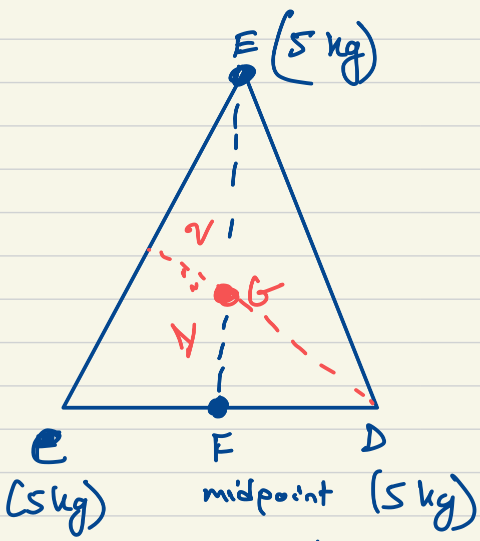
think of the entire mass to be at F

$6 \times 5 = 3 \times 10$

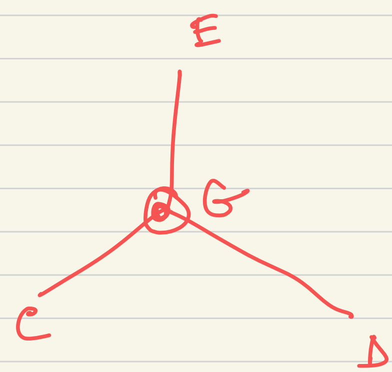
$30 = 30$

Another way to think of centroid

↑ centre of mass



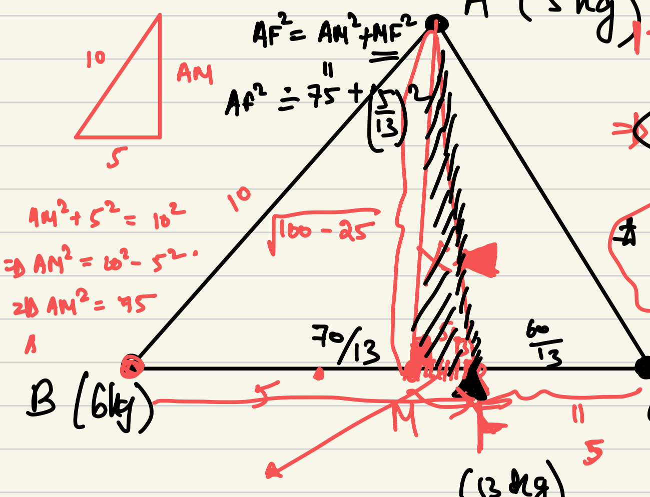
that makes [EF as the median]



$10 - \frac{60}{13} = BF$ $\qquad AX \cdot 5 = FX \cdot 13$

Challenge problem

centre of mass of the triangle



$AF^2 = AM^2 + MF^2$

$AF^2 = 75 + \left(\frac{5}{13}\right)^2$

$AM^2 + 5^2 = 10^2$

$\Rightarrow AM^2 = 10^2 - 5^2$

$\Rightarrow AM^2 = 75$

$\sqrt{100-25}$

$5 - \frac{60}{13} = \frac{65-60}{13} = 5/13$

$1 + \frac{AX}{FX} = \frac{13}{5} + 1$

$\Rightarrow \frac{FX + AX}{FX} = \frac{18}{5}$ (AF)

$\frac{\sqrt{75 + \frac{25}{169}}}{FX} = 18/5$

$5 \times \sqrt{75 + \frac{25}{169}} = FX$ (18)

$7 \times CF = 6 \times BF$

$1 + \frac{7}{6} = \frac{BF}{CF} + 1$

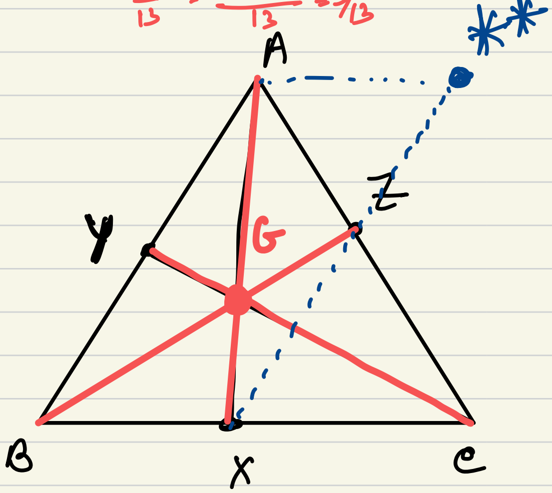
$\Rightarrow \frac{13}{6} = \frac{BF + CF}{CF} = \frac{10}{CF}$

$BC = 10\,cm$

$AB = 10\,cm$

$AC = 10\,cm$

Construction



create another triangle using med lines (medians)

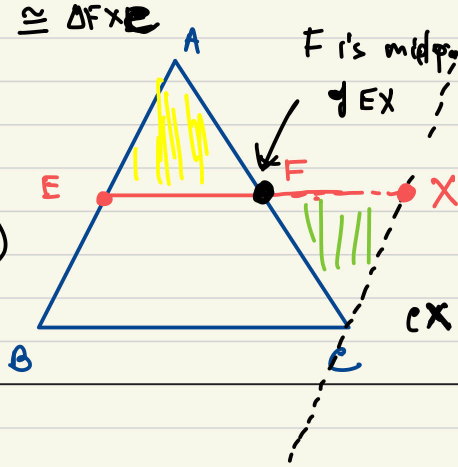
concurrency

||

passing through same point

Question Construct a new triangle whose side lengths are equal to the length of medians.

$\Delta AEF \cong \Delta FXC$



F is midpoint of EX

Given data! E is midpoint of AB

F is midpoint of AC

claim! $EF \parallel BC$

$EF = \frac{1}{2} BC$

Midpoint Theorem

$AF = FC$ (F is midpoint)

$\angle AFE = \angle XFC$ (vertically opp)

$\angle AEF = \angle FXC$ (alternate angle)

hint construct through C a line parallel to AB

$CX = EA$

||

BE

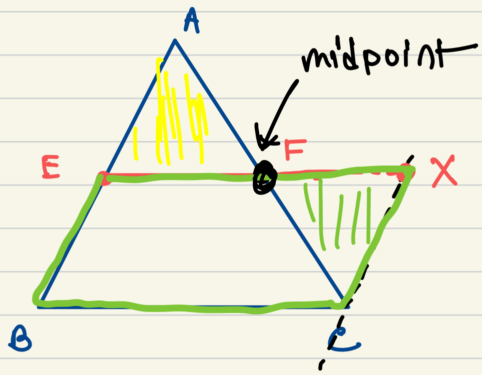
produce EF to meet black line at X

Subtle point

equiangular implies sides are proportional

$\Downarrow$

depends on midpoint theorem



$$\begin{cases} CX = BE \\ CX \parallel BE \end{cases}$$

H/W

if in a quadrilateral one pair of opposite sides are equal and parallel then its a $\parallel^{gm}$.

$\Rightarrow EX \parallel BC$

$\Rightarrow EF \parallel BC$

$EX = BC.$

$\parallel$

$2EF$

$EF = \frac{1}{2} BC$

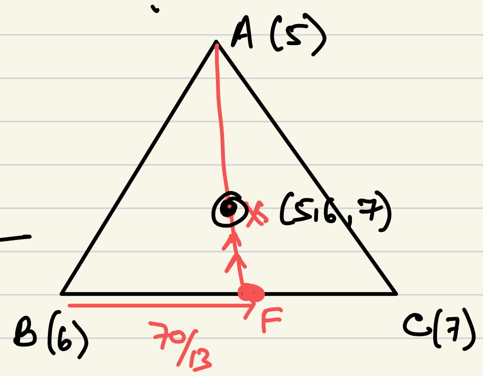
** We know how to visit the point X

Similarly

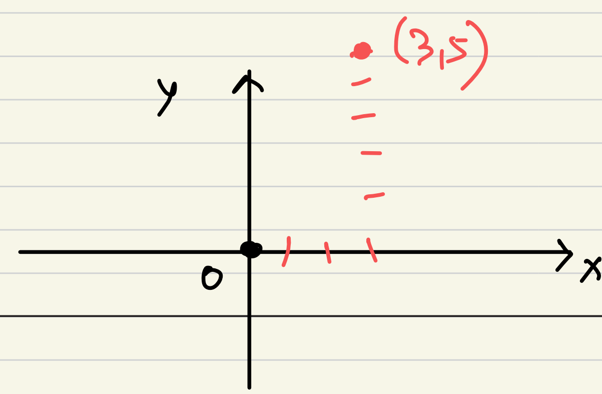
$$FX = \frac{\sqrt{75 + \frac{25}{169}}}{18}$$

do not calculate lengths

↳ work with ratios.



define (5, 6, 7) as the mass point coordinate of X w.r.t ΔABC



Visit point (3,5)

↑

code for location of a point