Another World of Geometry - 1

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Think on these problems.

Problem 1 - About mass point coordinates

Suppose ABC is a triangle. D, E, F are the midpoints of BC, CA, AB respectively. The line segments AD, BE and CF are known as *medians* of the triangle.

It is kind of surprising that that the medians of a triangle pass through a single point (three line segments may not always pass through a single point). We usually label this point as G. This is a **special point** inside a triangle. It is known as *centroid*.

Recall that the mass point coordinate of centroid of a triangle is (a, a, a) where a can be any positive integer. We discussed in detail what mass point coordinates are.

1. What happens if we slightly change one of the three mass point coordinates of the centroid? For example, can you draw the point $(a, a, a + 1)$.

2. If we allow non-integer mass point coordinates and change one of the coordinates slightly, say $(a, a, a + 0.1)$. what happens then?

Problem 2 - About centroid

We proved the midpoint theorem in class. It says, if ABC is any triangle, line segment joining midpoints of two sides of a triangle is parallel to and half of the third side.

Can you use the mid point theorem (and the fact that diagonals of a parallelogram bisect each other), to show that three medians of a triangle pass through a single point?

Problem 3 - About that triangle made of medians

In class, we showed that the medians of a triangle can be used to produce another triangle. Clearly write down the proof of this beautiful result.

Can you use vector algebra to prove this? In next session we will briefly introduce the idea of vector algebra (and how to use it to solve geometry problems). However here are a few nuggets that you may use right away.

1. Suppose A is the coordinate of the point A. This is a bit weird. But mathematicians do this sort of weird stuff all the time. We could have said $A = (x_1, y_1)$ but that requires us to write more. Hence we just write A. At once, it is the label of the point and a label for the coordinate of the point.

2. Midpoint of A and B is simply $\frac{A+B}{2}$. Can you rigorously prove this?

3. If A and B are two points in the plane, then the *directed* segment from A to B can be represented by the point $B - A$. That is if $A = (3, 4)$ and $B = (2, 1)$ then the directed segment from A to B can be represented by a single point $B - A = (2 - 3, 1 - 4) = (-1, -3)$. Can you say why this makes sense? After all we are thinking about 'arrows' or 'segments' as 'points'! This is the key idea of vector algebra and geometry.

4. Three directed segments XY, YZ and ZX is a triangle if their sum is 0. Why is this true? (Notice that this 'sum' makes sense because you can write XY as $Y - X$ etc. and then you can add those points.)

Problem 4 - about constructions

In class we talked about constructions using compass and straight edge (un-marked ruler). In particular we proved how to construct midpoint of a line segment.

1. Find a compass - straight edge construction that splits a line segment into 5 equal parts.

2. Find a compass - straight edge construction that computes square root of product of lengths of two given line segments.